Write the definition of an ellipse. **Use complete sentences and proper English as shown in lecture.** SCORE: ___ / 4 PTS

AN ELLIPSE IS THE LOCUS OF POINTS IN THE PLANE WHOSE DISTANCES TO TWO FIXED POINTS (FOCI) ADD UP TO A FIXED CONSTANT

Chris and Hunter live in the same town (in different houses). There is a road in that town such that, no matter where **SCORE:** ___ / 2 PTS
you are on the road, you are 2 miles closer to Chris's house than to Hunter's house. What is the shape of that road ?

P ROAD

$PH - PC = 2$

HYPERBOLA ②

H C

Using the definition of a hyperbola, find the equation of the hyperbola such that the distances from any point on the hyperbola to the foci $(0, \pm 6)$ differ by 4.

IF (x, y) IS ON THE HYPERBOLA

① $\sqrt{x^2 + (y+6)^2} - \sqrt{x^2 + (y-6)^2} = \pm 4$

① $\sqrt{x^2 + (y+6)^2} = \sqrt{x^2 + (y-6)^2} \pm 4$

① $x^2 + y^2 + 12y + 36 = x^2 + y^2 - 12y + 36 \pm 8\sqrt{x^2 + (y-6)^2} + 16$

① $24y - 16 = \pm 8\sqrt{x^2 + (y-6)^2}$

① $3y - 2 = \pm\sqrt{x^2 + (y-6)^2}$

① $9y^2 - 12y + 4 = x^2 + y^2 - 12y + 36$

① $8y^2 - x^2 = 32$

① $\dfrac{y^2}{4} - \dfrac{x^2}{32} = 1$

Find the standard form of the equation of the ellipse with foci $(-4, 5)$ and $(-4, -1)$ and a minor axis of length 12.
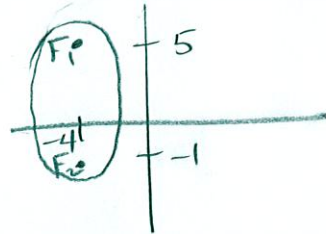
$$\text{CENTER} = \left(-4, \frac{5+-1}{2}\right) = (-4, 2) \quad \tfrac{1}{2}$$

$$C = 5 - 2 = 3 \quad \tfrac{1}{2}$$

$$2b = 12 \longrightarrow b = 6 \quad ①$$

$$a^2 = 6^2 + 3^2 = 45 \quad ①$$

$$\frac{(x+4)^2}{36} + \frac{(y-2)^2}{45} = 1$$

$① \quad \tfrac{1}{2} \quad\quad ① \quad \tfrac{1}{2}$

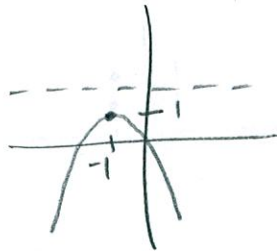Find the focus and directrix of the parabola with equation $y = -\frac{1}{2}x^2 - x + \frac{1}{2}$.

$-2y = x^2 + 2x - 1$

$-2y + 1 = x^2 + 2x$

$-2y + 2 = x^2 + 2x + 1$ ②

$(x + 1)^2 = -2(y - 1)$ ①

$4p = -2 \rightarrow p = -\frac{1}{2}$ ①

VERTEX $= (-1, 1)$

FOCUS $= (-1, 1 - \frac{1}{2})$

$= (-1, \frac{1}{2})$ ①

DIRECTRIX

$y = 1 + \frac{1}{2}$

① $y = \frac{3}{2}$ ①

Consider the ellipse with equation $4x^2 + y^2 + 8x - 10y + 13 = 0$.

SCORE: ___ / 8 PTS

[a] Find the standard form of the equation of the ellipse.

$$4x^2 + 8x \qquad + y^2 - 10y \qquad = -13$$

$$4(x^2 + 2x + 1) + (y^2 - 10y + 25) = -13 + 4 \cdot 1 + 25$$

$$4(x+1)^2 + (y-5)^2 = 16$$

$$\frac{(x+1)^2}{4} + \frac{(y-5)^2}{16} = 1$$

[b] Find the foci of the ellipse.

$$\text{CENTER} = (-1, 5)$$

$$16 = 4 + c^2$$

$$c = \sqrt{12} = 2\sqrt{3}$$

$$\text{FOCI} = (-1, 5 \pm 2\sqrt{3})$$